# Forecasting sovereign bond spreads with macroeconomic news sentiment 

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October 17, 2017


#### Abstract

We analyse sovereign bond spreads from Germany and enhance their modelling and prediction through macroeconomic news sentiment. Sentiment time series are created which mirror the mood in news regarding political and economical issues in European countries. Positive and negative sentiment is analysed separately taking into account market restrictions and trading venues. We are able to enhance the forecast errors in ARIMAX models through incorporating news sentiment series. Credit risk of sovereign bonds is therefore monitored more efficiently when news sentiment are taken into account.


Keywords Time series model, Bond spreads, News sentiment, ARIMAX, Credit risk

## Contents

1 Introduction ..... 3
2 Data ..... 4
2.1 Bond data ..... 4
2.2 Macroeconomic news sentiment ..... 4
3 Model ..... 5
4 Empirical results for long-term bonds ..... 6
4.1 Correlation with news time series ..... 7
4.2 Linear regression ..... 7
4.3 ARIMAX model ..... 10
5 Empirical results for short-term bonds ..... 14
5.1 Correlation with news time series ..... 14
5.2 Bubill examples - correlation and ARIMAX models ..... 16
6 Correlation over time ..... 21
7 Conclusion ..... 23
References ..... 25
Appendix ..... 28

## 1 Introduction

In the wake of the sovereign debt crisis in Europe, managing and monitoring credit risk arising from sovereign bonds is increasingly important. European countries have undergone changes in terms of their financial stability, and credit spreads have widened due to increased financial risk. Modelling of sovereign bond spreads is often linked to various macroeconomic factors such as the countries' GDP growth rate or inflation. These macroeconomic factors are monitored via scheduled announcements from official bodies e.g. treasuries and national banks but are also covered in news articles and unscheduled announcements. Changes in country dynamics and risks are reported and captured in news, which are classified as "macroeconomic news", and can be closely monitored and quantified through news sentiment analysis.

News sentiment for equities and in particular its use in equity trading has been widely covered in various studies over the last years. An overview of equity modelling and predictability enhancements through news sentiment is given in Mitra and Mitra [14]. The dynamics of asset prices, in particular their volatility is clearly affected by news events. These events are classified and quantified, news sentiment can be utilised to enhance volatility prediction (see e.g. Mitra et al [15]). Sentiment Analysis is used to improve trading decisions in equity markets. Firm-specific news sentiment affects the predicted asset return distribution; taking into consideration sentiment values increases the accuracy of the forecast and contributes to improved portfolio decisions as discussed in [12] and [19], amongst others. In the Fixed Income market however, news sentiment and its potential influence to bond spreads has just recently become more relevant in the light of electronification of bond trading (Lech et al [8]) and lacks thorough investigation. Especially macroeconomic news sentiment for sovereign bond spreads but also firm-specific news sentiment for corporate bond spreads can add value to both monitoring and forecasting of bonds. In this paper, we aim to fill this current gap and provide an extensive study on effects of news sentiment to bond spread predictions. In particular, we investigate the influence of macroeconomic news sentiment on bond spreads and develop a method to improve prediction and monitoring of sovereign spreads.

When analyzing bond spreads of European countries, various studies (e.g. [3], [5] and [13]) found influencing international and country-specific risk factors such as government debt. and characterised market dynamics such as liquidity issues and fiscal policies to effect bond spreads. Economic fundamentals are seen as drivers for sovereign spreads (see Dewachter et al.[6]); they have been utilised to explain yield spread movements and a significant effect has been found. Following a study by Afonso et al. [1], factors that influence sovereign spreads in Europe are time varying. The authors highlight the fact that financial determinants have changing effects on spreads, but that their influence is increasing in times of crisis. A further investigation of time-varying factors can by done by considering macroeconomic news, which report on changing dynamics and influences from issuing and neighbouring countries. News and sentiments for sovereign bond spreads were investigated by [16] and [4], amongst other. They investigated the influence of news announcements on spreads during the European debt crisis and found evidence, that information from government statements as well news from a European newsflash platform influenced yield spreads both nationally but also across countries, pointing to spill-over effects in the debt crisis.

Our paper contributes to the current literature an in-depth analysis of the impact

Table 1: Bond description of analysed Bunds

| Bond | First observed date | Days to Maturity | Maturity date | Coupon |
| :--- | :---: | :---: | :---: | :---: |
| Spread 1 | $2007-05-02$ | 3716 | $07 / 04 / 17$ | 4.25 |
| Spread 2 | $2008-05-30$ | 3687 | $07 / 04 / 18$ | 4.25 |
| Spread 3 | $2009-10-27$ | 3537 | $07 / 04 / 19$ | 3.5 |
| Spread 4 | $2010-05-05$ | 3713 | $07 / 04 / 20$ | 3 |
| Spread 5 | $2011-05-10$ | 3708 | $07 / 04 / 21$ | 3.25 |
| Spread 6 | $2012-04-12$ | 3735 | $07 / 04 / 22$ | 1.75 |
| Spread 7 | $2013-09-13$ | 3623 | $08 / 15 / 23$ | 2 |
| Spread 8 | $2014-09-11$ | 3626 | $08 / 15 / 24$ | 1 |
| Spread 9 | $2015-07-16$ | 3683 | $08 / 15 / 25$ | 1 |
| Spread 10 | $2016-07-13$ | 3685 | $08 / 15 / 26$ | 0 |
| Spread 11 | $2007-01-05$ | 11868 | $07 / 04 / 39$ | 4.25 |
| Spread 12 | $2008-08-11$ | 11650 | $07 / 04 / 40$ | 4.75 |
| Spread 13 | $2010-08-02$ | 11659 | $07 / 04 / 42$ | 3.25 |
| Spread 14 | $2012-04-26$ | 11757 | $07 / 04 / 44$ | 2.5 |
| Spread 15 | $2007-01-05$ | 11857 | $08 / 15 / 46$ | 2.5 |

of processed macroeconomic news and its sentiment towards European sovereign yield spreads. In particular, we investigate the dynamics of German Bubills and Bund spreads and find a relation between their forecasts and news sentiment time series. Our findings show that the forecast of yield spreads can be enhanced when daily news sentiment is taken into account. News is split into positive and negative news items, their influences are investigated separately as well as jointly in a multivariate ARIMAX set-up. We find that negative sentiment as well as the volume of incoming news lead to better one-step ahead predictions of spreads. We find significant correlations between sentiment time series and yield spreads and analyse these correlation overtime. Our findings support earlier results on time-varying factors, since also for news sentiment, correlations vary over time and have changing dynamics depending on the state of the market. We conclude that news sentiment adds value to modelling sovereign yield spreads and should be taken into account when analyzing and monitoring spreads.

## 2 Data

### 2.1 Bond data

We analyze long- and short-term bonds issued by Germany in this study. During the Eurozone debt crisis, German bonds were considered as the "safe haven", often referred to as the "riskless" asset. We analyse in our study 36 Bubills (short-term bonds) and 15 Bunds (long-term bonds) issued from Germany between 2007 and 2017. We analyse time series data from Thomson Reuters' Datascope and calculate spreads between the bond yields and the AAA-rated bond yield quoted from the European Central Bank (ECB).

### 2.2 Macroeconomic news sentiment

We wish to analyse the effect news articles and announcements have on bond yields. In our study, macroeconomic sentiment comprised by RavenPack is employed. RavenPack marks every news item that arises from various sources with a sentiment value. This sentiment value lies between -1 and 1 and quantifies the
sentiment of a particular news item for the chosen entity. In our case, we choose the bond issuer as the entity we would like to follow. Out of all sentiment values that stream in over the day, we create daily news time series. The news time series are all based on RavenPack's Macroeconomic News Sentiment.

For our particular experiment in this paper, we follow macroeconomic news, which are bundled under the key words for Germany, namely "Germany" and "Government Germany", representing the issuer of the bonds. A typical macroeconomic news example from our database includes the time stamp, a relevance of the news with respect to the key word as well as the sentiment value ("ess").

We create nine different time series based on the relevance and sentiment value we receive from RavenPack's database to build daily news sentiment values which can be utilized as an input variable for our time series models. Firstly, we split the sentiment values into two sub-categories handling positive and negative newssentiment separately. We conduct a pre-analysis of our news sentiment data which allows us to consider all news after market close time until market close time on the following day for the daily news sentiment. We create

1. a mean new-sentiment value time series
2. a number of news time series
3. a news-impact time series
for the three categories
a. all news
b. positive news
c. negative news

Therefore, we create nine different time series observed throughout the time interval where the bond is active. All news time series are utilized as regressors in a regression model as well as external variables in an ARIMA model. Furthermore their correlation with the yield spread is calculated for the whole time period as well as in a rolling window.

## 3 Model

In order to establish whether a relation between the different news time series and the yield spread series exists, we test for correlation between the daily yield spread series and all nine news time series. We calculate Pearson's correlation between the daily time series and test whether the correlation is significant. Furthermore, the correlation is observed within a rolling window to see time-varying features of the correlation between time series.

Secondly, a linear regression is performed to analyse the effects of news time series on the yield spreads. All nine news time series are taken as regressors in a variety of combinations. We report here results for regression with three news series regressors.

Lastly, we apply an Integrated Autoregressive Moving Average (ARIMA) model to analyse and forecast bonds yields. We additionally add external explanatory
variables to the model, therefore fitting an $\operatorname{ARIMAX}(\mathrm{p}, \mathrm{i}, \mathrm{q})$ model to yield spreads. The ARIMAX ( $\mathrm{p}, \mathrm{i}, \mathrm{q}$ ) model is given through

$$
\begin{equation*}
d_{t}=\phi_{0}+\sum_{k=1}^{p} \phi_{k} d_{t-k}+a_{t}+\sum_{k=1}^{q} \theta_{k} a_{t-k}+\sum_{l=1}^{m} x_{l t} \tag{1}
\end{equation*}
$$

where $d_{t}$ is the i-th differenced series of the time series $r_{t},\left\{a_{t}\right\}$ is a white noise series and $x_{l t}$ is the $l$-th external explanatory variable, $l=1, \ldots, m$. The explanatory variable are uni- or multivariate. We model the first difference of our time series, therefore $i=1$. An ARIMAX model was also successfully applied by Apergis [2] to analyse CDS spreads and newswire sentiments. His study results in improved forecast errors when external news time series were allowed. We model the yield spreads firstly with an ARIMA(p,1,q) model and compare the resulting in-sample and out-of-sample one-step ahead forecast errors to those which arise from $\operatorname{ARIMAX}(p, 1, q)$ model with various external regressors. We run a considerable amount of models on our daily yield spread series, taking into account uni- as well as multivariate external explanatory variables. We can improve the forecast errors throughout all analysed bonds when sentiment is taken into consideration. This points to the fact that news sentiment has value for bond yield modelling and risk assessment. Monitoring macroeconomic news sentiment series in addition to the actual yield spread can lead to early warning signs for unexpected changes in yields or structural changes visible in the yield spreads.

## 4 Empirical results for long-term bonds

Yields and spreads of "Bundesanleihen", national bonds emitted by the Federal Republic of Germany, are affected by various internal and external factors. Bundesanleihen express expectations about inflation and economic growth but likewise depend on numerous determinants that cannot be isolated explicitly. These other factors might well be captured through news sentiment time series. In the following we would like to determine whether these sentiment series can add value to regression analysis and bond spread forecasts through an ARIMAX model.

We firstly analyze long-term loans emitted by Germany, the so-called Bunds. In total, we analyze 15 instruments with a maturity between 5 and 30 years. The AAA-rated European bond is chosen as a benchmark, therefore the spread series which we model is created as a spread with ECB AAA Svenson yields. For all 15 loans, we perform experiments with news-sentiment time series and model the spread time series by including this information from our news sources. Firstly, the correlation between the spread series and nine different news-related time series are estimated and its significance checked.

In addition, we find an appropriate ARIMA order for the spread series'. Our conducted tests calculate the Akaike information criterion and reveal that ARIMA $(2,1,2)$ is an appropriate model order for a typical spread series from the Bunds.

Furthermore, we conduct a unit root test (Augmentend Dickey Fuller) to see whether the time series is non-stationary and differencing is necessary. The nullhypothesis is that of non-stationarity, therefore a small p-value (less than $5 \%$ ) points to a stationary time series, the null-hypothesis of non-stationarity can be rejected. A second unit root test is the KPSS test (Kwiatkowski-Phillips-Schmidt-Shin), where the null-hypothesis is that of stationarity. For our bond data sets, a small p-value is reported, so stationarity can be rejected.

In the following, we will analyse 15 Bunds and report the results of a correlation test, a linear regression and the ARIMAX model. We firstly state the significance of correlation between the spread series and the nine news-related time series. The second result for each Bund shows the summary statistics for a linear regression, whereby the number of all news, the positive impact and the negative impact time series were chosen regressors. The choice of these regressors is the results of a variety of regression analysis with changing regressors. This combination is most suitable for a majority of Bunds.

Lastly we perform ARIMA modelling of the instruments. The ARIMAX (2,1,2) models were fitted in an in-sample period and the one-step ahead forecast was further evaluated in an out-of-sample window. For all Bunds, we analysed different news time series as external variables and show here the results for the most promising model set-ups. We show error measures for a one-step ahead forecast in the in-sample as well as out-of-sample window and distinguish between eight model set-ups. Our ARIMAX models have the following external regressors:

1. no external regressor
2. Number of all news; All News Impact; Number of positive news; Positive news impact
3. Number of all news; All News Impact; Number of negative news; Negative news impact
4. Number of all news; All News Impact
5. Positive Impact; Negative Impact
6. Mean Positive Sentiment
7. All News Impact
8. Number of all news

The estimated models cover both multi- as well as univariate external variables. The models in this final analysis were chosen from a larger set of univariate and multivariate model set-ups and represent the most promising forecast models for these bonds.

### 4.1 Correlation with news time series

The correlation analysis shows that we find a significant correlation between the spread time series and the news sentiment time series for most cases . Table 4.1 shows the percentages of bonds with significant correlations for each sentiment time series with spread and squared spread time series. In $87 \%$ of analysed spread time series, at least one news sentiment series showed significant correlation with the spread series.

### 4.2 Linear regression

A linear regression was performed on all 15 bunds, where the number of all news, the positive impact series and the negative impact series were the chosen regressors. We report here the summary statistics as well as the diagnostic plots and find significant

Table 2: Percentage of significant correlations between spread and sentiment time series'

| News time series | Spread | Squared spread |
| :--- | :---: | :---: |
| All Sentiment | $40 \%$ | $40 \%$ |
| Nr all news | $73 \%$ | $73 \%$ |
| All impact | $33 \%$ | $33 \%$ |
| Positive Sentiment | $20 \%$ | $13 \%$ |
| Nr positive news | $80 \%$ | $73 \%$ |
| Positive impact | $20 \%$ | $13 \%$ |
| Negative Sentiment | $60 \%$ | $67 \%$ |
| Nr negative news | $67 \%$ | $73 \%$ |
| Negative impact | $60 \%$ | $67 \%$ |

regressors for most of the spreads, supporting the fact that sentiment information plays a role in explaining bond spreads. Significant regressors for the first bond are the number of all news as well as the negative impact series. The diagnostics plots show a more or less vertical plot for the residuals, therefore the residuals so not exhibit any trend that could be captured further.

The second bond with a duration of 10 years chooses all three regressors as significant. Again, diagnostic plots do not show any trends or outliers in the residuals that would have to be removed.

Spread Nr. 3 also chooses all three regressors as significant. Positive impact has a negative coefficient here, opposite to the coefficient for the second analysed spread.

All three regressors are again significant for the fourth analysed spread, which has a duration of 10 years with a start date of May 2010.

The regression analysis plots for Spread 5 show no trends in the residuals.
Spread 6 does not identify the positive news impact series as a significant regressors, but number of all news and negative impact are chosen.

The number of all news is the only significant regressor for Spread 7. Again, for Spread 8, just the number of news is identified as significant, the diagnostic plot show some deviation from the normal distribution.

For Spread 9 and 10 , neither of the regressors is flagged as being significant.
Spread 11 exhibits once again significant regressors from the "number of all news".

The linear regression analysis for spread 12 chooses the Number of All News as well as the Negative Impact time series as regressors.

The same analysis is valid for Spread 13, the number of all news as well as Negative Impact is chosen.

Bund Spread time series: 1

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1062 | 0.0150 | -7.10 | 0.0000 |
| NrOfAllNews | -0.0003 | 0.0001 | -3.48 | 0.0005 |
| PosImpact | 0.0329 | 0.0261 | 1.26 | 0.2070 |
| NegImpact | 0.1904 | 0.0206 | 9.24 | 0.0000 |



Table 3: Summary and diagnostic plots for regression analysis

Again, the same regressors are chosen for Spread 14. The last spread exhibits significant correlations with the negative sentiment time series.

Bund Spread time series: 2

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1647 | 0.0185 | -8.91 | 0.0000 |
| NrOfAllNews | -0.0002 | 0.0001 | -2.07 | 0.0390 |
| PosImpact | 0.0674 | 0.0320 | 2.11 | 0.0352 |
| NegImpact | 0.2242 | 0.0248 | 9.02 | 0.0000 |



Table 4: Summary and Diagnostic plots for regression analysis

### 4.3 ARIMAX model

We model the 15 bunds which we analyse through an ARIMAX ( $2,1,2$ ) model. The forecast errors of the pure $\operatorname{ARIMA}(2,1,2)$ model are compared to those of the extended ARIMAX model. For in- and out-of-sample one-step ahead forecasts, we consider the error measures Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Scaled Error (MASE). We analyse for each bond an in-sample period, which covers $85 \%$ of the length of the time series, and an out-ofsample period, which is the remaining $15 \%$ of length of the time series.

Below, the results for the ARIMAX $(2,1,2)$ models are plotted. Error measure tables for all Bunds are stated in Appendix 7. For Spread 1, in the one-step ahead forecast within the in-sample period, model 3 gives the best results, in the outof sample period, bet results are achieved for model 6 . Spread 2 finds the best ARIMAX model for the in-sample period is model 2 and 4, whereas the best model in the out-of sample period is Model 7 and 8. There is therefore not a single best suited model, since nearly all sentiment series show significant correlation with the spread series, various regressors and external variables can be chosen and add value to the model forecast. Best ARIMAX models for Spread 3 are the third one for the in-sample period and the first one for the out-of-sample period. This means that in this particular case, adding news sentiment data does not add extra value to the ARIMAX model.

The best forecast ARIMAX model for Spread 4 is model 3 and 8 , various com-

Bund Spread time series: 3

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.0988 | 0.0212 | -4.66 | 0.0000 |
| NrOfAllNews | -0.0003 | 0.0001 | -3.34 | 0.0009 |
| PosImpact | -0.1254 | 0.0389 | -3.22 | 0.0013 |
| NegImpact | 0.1583 | 0.0262 | 6.05 | 0.0000 |



Table 5: Summary and Diagnostic plots for regression analysis
binations of external explanatory variables can be chosen to achieve better error measures. The best ARIMAX model for spread 5 is model 3 for the in-sample period and model 7 for the out-of-sample period. Best ARIMAX models for Spread 6 are models 6 and 8 .

Best ARIMAX models are model 6 and 7 for Spread 7. Model 4 and 6 are the best choice for the ARIMAX model for Spread 8. Spread 9 is rather uncommon. Neither of the regressors is flagged as being significant. However, the ARIMAX one-step ahead forecast is improved for Spread 9 when the sentiment data is added.

Best ARIMAX models for Spread 12 are models 7 and 8. However, the best ARIMAX models for Spread 13 are models 2 and 7 . Model 2 (in-sample) and model 4 (out-of sample) lead to the smallest one-step ahead forecast errors for Spread 14. In line with the regression analysis, the regressor "Negative Impact" is significant for Spread 15. Forecast errors are smallest for models and 1, making it the second spread to prioritise a simple ARIMA $(2,1,2)$ model over a ARIMAX model with external explanatory variables.

Overall, the best performing ARIMAX model over these bunds spread time series are Model 2 and 3 for in-sample and Model 7 for out-of sample one-step ahead predictions. Therefore, multivariate models with Number of all news, All News Impact, Number of positive news, Positive news impact or Number of all news, All News Impact Number of negative news Negative news impact add the most value to the ARIMAX model for in-sample forecast. Out-of-sample forecast is best in univariate settings. Here, choosing Mean Positive Sentiment or All News

Figure 1: Out-of-sample 1-step ahead forecast for Spread 1


Figure 2: Out-of-sample 1-step ahead forecast for Spread 2


Figure 3: Out-of-sample 1-step ahead forecast for Spread 3


Figure 4: Out-of-sample 1-step ahead forecast for Spread 4


Figure 5: Out-of-sample 1-step ahead forecast for Spread 5


Figure 6: Out-of-sample 1-step ahead forecast for Spread 6


Figure 7: Out-of-sample 1-step ahead forecast for Spread 7


Figure 8: Out-of-sample 1-step ahead forecast for Spread 8


Figure 9: Out-of-sample 1-step ahead forecast for Spread 9


Figure 10: Out-of-sample 1-step ahead forecast for Spread 10



Figure 11: Out-of-sample 1-step ahead forecast for Spread 11


Figure 12: Out-of-sample 1-step ahead forecast for Spread 12


| Bund Spread time series: 4 |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| (Intercept) | -0.1141 | 0.0233 | -4.91 | 0.0000 |
| NrOfAllNews | -0.0003 | 0.0001 | -3.40 | 0.0007 |
| PosImpact | -0.1243 | 0.0428 | -2.90 | 0.0037 |
| NegImpact | 0.1784 | 0.0288 | 6.20 | 0.0000 |



Table 6: Summmary and Diagnostic plots for regression analysis

Impact brings the best results. All these models outperform the simple ARIMA model without external regressor in terms of the chosen error measures.

## 5 Empirical results for short-term bonds

The following analysis concentrates on short-term bonds, Bubills, issued from the Federal Republic of Germany. We analyse the spreads of these Bubills which were active between 2007 and 2017 and utilize the aforementioned news time series from Section 2 to model the spreads.

### 5.1 Correlation with news time series

We start by analyzing the correlation between the spreads of Bubills to ECB AAArated rates and the news time series. To create news time series', we observe news for "Government of Germany" and include all news sentiment items above a relevance of 60 . We tested the percentage of spread series showing a significant correlation with at least one of the news time series. Table 5.1 shows the percentages of spreads with significant correlations with news time series, where the cut-off point for relevance was varied between 30 and 90 . A similar picture emerges when we observe news from topic "Germany".

We further distinguish between all news items as well as news items from categories "economics", "politics" and "business". We analyse the correlation between

Bund Spread time series: 5

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1246 | 0.0256 | -4.87 | 0.0000 |
| NrOfAllNews | -0.0004 | 0.0001 | -3.44 | 0.0006 |
| PosImpact | -0.0952 | 0.0474 | -2.01 | 0.0448 |
| NegImpact | 0.2044 | 0.0321 | 6.36 | 0.0000 |



Table 7: Summary and Diagnostic plots for regression analysis
these news time series and several spread time series: for each bond, we create the spread series, the first difference spread time series as well as the squared spread time series, which serves as a proxy for daily volatility.

The results show significant correlations mainly between the squared spreads and the news time series, followed by significant correlations between spreads and news time series'. Analysing squared spreads, we find the highest number of significant correlations with the news time series "All Sentiment" and "Negative Sentiment". Here, significant correlations can be found in around $25 \%$ of cases. Similar correlations can be found for the spread time series itself. The highest number of significant correlations can be seen with the number of negative news time series, whereas "All Sentiment", "Positive Sentiment" and "Negative Sentiment" time series show a similar proportion of significant correlations. It has to be noted that the percentage of Bubill bond spreads series showing significant correlation with news time series is lower than that of long-term bond spreads over the same time period. Bubill spreads often show a sharp increase in volatility over the last weeks or month before maturity. Especially these time intervals are less likely to exhibit significant correlation with news time series.

In the following, we utilize the news category covering the entity "Government of Germany" and test its influence on the German Bubills. We would like to analyze whether news classified as "governmental" have a stronger impact on Bubill spreads. Again, we analyze pure spreads as well as squared spreads and conduct correlation tests, regression analysis as well as one-step-ahead forecasts through an

Bund Spread time series: 6

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1289 | 0.0192 | -6.73 | 0.0000 |
| NrOfAllNews | -0.0003 | 0.0001 | -3.37 | 0.0008 |
| PosImpact | -0.0320 | 0.0355 | -0.90 | 0.3673 |
| NegImpact | 0.1277 | 0.0246 | 5.20 | 0.0000 |



Table 8: Summary and Diagnostic plots for regression analysis

ARIMA $(1,1,1)$ model with external regressors. We analyze the effect of news sentiment with a relevance $>60$. Our results show significant influences from news time series associated to "Government Germany" to both pure and squared spreads. In addition, both positive and negative sentiment time series seem to have an effect on the spreads. However, rarely all news time series show significant correlations, typically either time series regarding all and positive or time series regarding all and negative sentiment have significant correlation. Whether positive or negative news series are significant is thought to be due to the business cycle state the analysed Bubill falls under, meaning that in times of recession negative news have a higher impact than positive news and vice versa.

### 5.2 Bubill examples - correlation and ARIMAX models

The following example shows the spread of a zero-coupon Bubill issued on 25th September 2009 with a duration of 110 days until January 2010 where significant correlation with negative news time series can be observed. We firstly depict the spread time series as well as the all, positive and negative sentiment time series over the duration of the bond.

Forecasts through the ARIMAX-model with various uni- and multivariate external variables lead to results stated in Figure 5.2. The chosen ARIMA order is here ( $1,1,1$ ), which was again determined through the Akaike Information criterion. The graphs of the forecasts show a close forecast, the error analysis points to

Figure 13: Out-of-sample 1-step ahead forecast for Spread 13



Figure 14: Out-of-sample 1-step ahead forecast for Spread 14



Figure 15: Out-of-sample 1-step ahead forecast for Spread 15


Figure 16: Spread and news time series for Bubill spread 1



P-value All Sentiment: 0.008656504 Correlation significant
P-value Nr all news 0.3328246
P-value All impact 0.001045843 Correlation significant
P-value Positive Sentiment 0.1791648
P-value Nr positive news 0.6119549
P-value Positive impact 0.1003025
P-value Negative Sentiment 0.0144835 Correlation significant
P-value Nr negative news 0.01172509 Correlation significant
P-value Negative impact 0.01388118 Correlation significant

Bund Spread time series: 7

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1309 | 0.0066 | -19.79 | 0.0000 |
| NrOfAllNews | 0.0001 | 0.0000 | 2.45 | 0.0144 |
| PosImpact | 0.0012 | 0.0124 | 0.10 | 0.9207 |
| NegImpact | 0.0050 | 0.0087 | 0.57 | 0.5677 |



Table 9: Summary and Diagnostic plots for regression analysis

ARIMAX-models 3 and 5 being the best models for this Bubill.
The next example shows a spread time series which exhibits significant correlation with the positive news sentiment time series. Furthermore, the regression analysis shows a significance for this explanatory variable. The ARIMA $(1,1,1)$ analysis and forecast highlights the fact that the external regressor improves the one-step ahead forecast which is computed in both settings, an in-sample and out-of-sample framework. The ARIMA with the lowest forecast errors are Model 2 and 8, which include the time series "Nr of all news", "Impact of all news", "Nr of positive news" and "Impact of positive news" as external regressors. The analysed zero-coupon Bubill is issued on $13 / 07 / 16$ with a duration of 182 days until $01 / 11 / 17$.

THese two examples highlight the fact, that correlations between spread and news time series vary over time, leading to different "best" external variables for the ARIMAX model. However, in all our examined cases, including the external variables in the ARIMAX models improved the one-step ahead forecast. We therefore conclude that including news sentiment in modelling spreads improves the forecast accuracy and gives valuable input to the forecast.

We analysed 36 Bubill spreads with issuing dates between 2007 and 2017. For all thee instruments, we analysed and tested the correlation, performed regression analysis and conducted one-step ahead ARIMAX prediction within eight different model set-ups. For our experiments, the best performing ARIMAX model is Model 3 , followed closely by Model 4. The chosen regressors in the analysed eight model set-ups are a combination of 1.)All Sentiment, 2.) Number Of All News 3.) All

Figure 17: 1-step ahead forecast: In-Sample ARIMA(1,1,1) modelling for Bubill spread 1


Figure 18: 1-step ahead forecast: Out-of-Sample ARIMA $(1,1,1)$ modelling for Bubill spread 1



Order ARIMA-Model: 111
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE
$0.030748240 .02198856-51.9101986 .057540 .9093398$
$0.029346090 .02147398-50.7211485 .420700 .8880590$
$0.029255270 .02048970-52.7325184 .604400 .8473543$
$0.030109340 .02156802-52.9817185 .149800 .8919482$
$0.030159910 .02169355-52.8018584 .788360 .8971396$
$0.030747530 .02199991-51.8329585 .996360 .9098092$
$0.030018160 .02172572-50.5019185 .132740 .8984698$
$0.030456300 .02221311-51.4132286 .164720 .9186261$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 3 & 7 & 3 & 3\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE 0.079403480 .0616746165 .86530115 .15560 .9251956 0.080131670 .0619409765 .66878116 .15460 .9291914 0.078115810 .0603945972 .72946116 .45770 .9059937 0.079087020 .0610527271 .98895115 .57680 .9158666 $0.078192380 .0599488971 .90847113 .6667 \quad 0.8993077$ 0.079425500 .0617140065 .82163115 .19240 .9257866 0.080548460 .0628585464 .11149117 .33790 .9429561 0.079050870 .0617422668 .06148117 .14620 .9262104 Out-of-sample: Model with lowest forecast error RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 5 & 7 & 5 & 5\end{array}$

Bund Spread time series: 8

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.0908 | 0.0049 | -18.45 | 0.0000 |
| NrOfAllNews | 0.0001 | 0.0000 | 2.35 | 0.0190 |
| PosImpact | 0.0065 | 0.0094 | 0.69 | 0.4898 |
| NegImpact | 0.0046 | 0.0064 | 0.72 | 0.4712 |



Table 10: Diagnostic plots for regression analysis

Impact, 4.) Positive Sentiment, 5.) Number Of Positive News, 6.) Positive Impact, 7.) Negative Sentiment, 8.) Number Of Negative News, 9.) Negative Impact. ARIMAX Models 1 to 8 are:

1. without regressor
2. Regressors $2,3,5,6$
3. Regressors $1,2,7,8$
4. Regressors $1,4,7$
5. Regressors 6,9
6. Regressors 4
7. Regressors 3
8. Regressors 2

Bund Spread time series: 9

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.0719 | 0.0062 | -11.55 | 0.0000 |
| NrOfAllNews | -0.0000 | 0.0000 | -0.31 | 0.7548 |
| PosImpact | -0.0041 | 0.0121 | -0.34 | 0.7354 |
| NegImpact | -0.0062 | 0.0086 | -0.72 | 0.4701 |






Table 11: Summary and Diagnostic plots for regression analysis

Our analysis resulted in the following percentages of best-performing ARIMAXModels with respect to analyse external regressors:

| Model | Fit | Forecast |
| :--- | ---: | ---: |
| 1 | $2 \%$ | $8 \%$ |
| 2 | $21 \%$ | $11 \%$ |
| 3 | $26 \%$ | $21 \%$ |
| 4 | $15 \%$ | $18 \%$ |
| 5 | $10 \%$ | $16 \%$ |
| 6 | $2 \%$ | $5 \%$ |
| 7 | $13 \%$ | $16 \%$ |
| 8 | $11 \%$ | $5 \%$ |

## 6 Correlation over time

In order to address changing dynamics of both spreads and news time series, we investigate, how the correlation between the spread series and the nine news time series is evolving over time. In particular, we plot rolling correlation for Bubills and Bunds, investigating two spread series in depth.

First, we would like to consider Bunds and their correlation with news time series aggregated from Raven Pack news for the entity "Germany" with a relevance of above 60 . The rolling correlation is calculated with a window size of 250 days. In Figures 6 and 6, we depict the evolution of the correlation between spread and

Bund Spread time series: 10

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.0024 | 0.0111 | -0.21 | 0.8312 |
| NrOfAllNews | -0.0000 | 0.0001 | -0.33 | 0.7416 |
| PosImpact | -0.0173 | 0.0203 | -0.85 | 0.3943 |
| NegImpact | -0.0111 | 0.0157 | -0.71 | 0.4777 |



Table 12: Diagnostic plots for regression analysis
i) average sentiment series, ii) news volume series and iii) impact series. We can clearly see, that the observed correlation changes over time for all three settings, running through periods with positive and negative correlations as well as periods with very low correlation between the spreads and the news time series.

The first example shows the evolution of a bond spread between 2011 and 2017. Most notable is the shift from positive to negative and back to positive correlation of the spread and the news time series "All Sentiment".

Secondly, we investigate the rolling correlations for Bubill spreads. The considered news entity is "Government Germany" and all news items with a relevance of above 60 are taken into account. Here, the rolling window size is 120 days, since the time series are typically shorter. The example in Figure 6 shows correlation for sentiment, volume and impact series. All plots exhibit changing correlation over time, but they remain relatively stable in the considered time frame. Positive sentiment and impact series have the highest positive correlation with the Bubill spread, whereas the negative volume series has the highest correlation from the considered volume series. This correlation undergoes a change, it increases over the second half of the time period.

Bund Spread time series: 11

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1850 | 0.0106 | -17.50 | 0.0000 |
| NrOfAllNews | -0.0004 | 0.0001 | -7.27 | 0.0000 |
| PosImpact | 0.0136 | 0.0184 | 0.74 | 0.4609 |
| NegImpact | 0.1130 | 0.0145 | 7.78 | 0.0000 |



Table 13: Summary and Diagnostic plots for regression analysis

## 7 Conclusion

Our analysis finds clear links between aggregated news time series and sovereign bond spreads. We investigate the behaviour of both long- and short-term bonds and find in most cases significant correlations between the spread time series as well as news time series, which take into account either the news sentiment or the volume of the news. We distinguish between all, positive and negative news items and found significant correlations between these series and the bond spread. Whether positive or negative news series showed a higher correlation might depend on the business cycle. We therefore recommend to take several sentiment series into account to cover various characteristics in changing markets.

Our analysis further showed that correlation and forecast errors clearly vary through time. We propose to monitor correlation changes over time to recognise changing market conditions as well as to identify relevant external regressors for a one-step ahead forecast. The ARIMAX models show enhanced error measures in both in-sample and out-of sample performance when news time series were taken into account. A multivariate model set-up utilizing All Sentiment, Number Of All News, Negative Sentiment and Number Of Negative News as regressors outperformed the other set-ups in terms of smallest forecasts errors.

Future work will cover an in-depth analysis of regressors and their influence on bond spreads. The instrument universe shall be broadened, in particular other countries shall be taken into account and further spreads shall be investigated. A

| Bund Spread time series: 12 |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| (Intercept) | -0.1926 | 0.0126 | -15.26 | 0.0000 |
| NrOfAllNews | -0.0003 | 0.0001 | -5.57 | 0.0000 |
| PosImpact | -0.0244 | 0.0220 | -1.11 | 0.2678 |
| NegImpact | 0.0808 | 0.0167 | 4.83 | 0.0000 |



Table 14: Summary and Diagnostic plots for regression analysis
first outlook confirmed the findings in this paper for other countries, an in-depth analysis will be considered in the near future.

## Acknowledgement:

This work is part of the project SENRISK E!10488 supported by funding from Eurostars2 joint programme with co-funding from the European Union Horizon 2020 research and innovation programme, which we gratefully acknowledge.

Bund Spread time series: 13

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1678 | 0.0162 | -10.37 | 0.0000 |
| NrOfAllNews | -0.0003 | 0.0001 | -4.23 | 0.0000 |
| PosImpact | -0.0368 | 0.0297 | -1.24 | 0.2154 |
| NegImpact | 0.1079 | 0.0202 | 5.35 | 0.0000 |






Table 15: Summary and Diagnostic plots for regression analysis

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Bund Spread time series: 14

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1510 | 0.0116 | -13.02 | 0.0000 |
| NrOfAllNews | -0.0002 | 0.0001 | -3.80 | 0.0001 |
| PosImpact | 0.0036 | 0.0215 | 0.17 | 0.8671 |
| NegImpact | 0.0303 | 0.0149 | 2.03 | 0.0422 |



Table 16: Diagnostic plots for regression analysis
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Bund Spread time series: 15

|  | Estimate | Std. Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| ---: | ---: | ---: | ---: | ---: |
| (Intercept) | -0.1287 | 0.0094 | -13.63 | 0.0000 |
| NrOfAllNews | 0.0001 | 0.0001 | 1.13 | 0.2605 |
| PosImpact | -0.0081 | 0.0177 | -0.46 | 0.6465 |
| NegImpact | -0.0314 | 0.0125 | -2.51 | 0.0122 |






Table 17: Summary and Diagnostic plots for regression analysis

| Relevance | $>30$ | $>40$ | $>50$ | $>60$ | $>70$ | $>80$ | $>90$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bond spreads with <br> sign. correlation | $61 \%$ | $50 \%$ | $56 \%$ | $61 \%$ | $61 \%$ | $42 \%$ | $33 \%$ |

Table 18: Percentage of significant correlation between short-term bond spreads and news time series
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Figure 19: Spread and news time series for Bubill spread 2



P-value All Sentiment: 0.7374913
P-value Nr all news 0.03652966 Correlation significant
P-value All impact 0.1707299
P-value Positive Sentiment 3.095744e-05 Correlation significant
P-value Nr positive news 0.2534435
P-value Positive impact 5.862235e-05 Correlation significant
P-value Negative Sentiment 0.2486
P-value Nr negative news 0.0541304
P-value Negative impact 0.2821196
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## Appendix

Error analysis of ARIMAX models for long-term bonds issued by Germany:

Figure 20: 1-step ahead forecast: In-Sample ARIMA(1,1,1) modelling for Bubill spread 2



Figure 21: 1-step ahead forecast: Out-of-Sample ARIMA(1,1,1) modelling for Bubill spread 2



Order ARIMA-Model: 111
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE
$0.038151930 .02767686-33.92700163 .92340 .8159254$
$0.037483490 .02727129-33.11175152 .74130 .8039693$
$0.036944600 .02721440-40.78214155 .41710 .8022919$
$0.037180250 .02737835-48.56116161 .06160 .8071252$
$0.037838610 .02722910-40.14470161 .67090 .8027254$
$0.038081330 .02753248-38.33215159 .71440 .8116691$
$0.038078680 .02770850-36.01472167 .55840 .8168582$
$0.037740940 .02713811-38.71573157 .55820 .8000431$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 8 & 2 & 2 & 8\end{array}$

Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
0.22511640 .167052360 .62028137 .69250 .8805567
0.22417890 .166584159 .79154139 .75250 .8780888
0.22403750 .165885758 .97553138 .58520 .8744073
0.22742100 .168736356 .24815140 .51850 .8894337
0.22489520 .165848060 .25344135 .52320 .8742089
0.22730810 .168633960 .11690138 .34200 .8888939
0.22376020 .166230762 .26870135 .18650 .8762261
0.22296570 .165669263 .17779136 .19970 .8732662

Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}8 & 8 & 4 & 7 & 8\end{array}$

Figure 22: Rolling correlation between Bunds and news


Figure 23: Rolling correlation between Bunds and news


Figure 24: Rolling correlation between Bubills and news


Figure 25: In-sample 1-step ahead forecast for Bund Spread 1


Figure 26: ARIMAX models: Forecast errors for Spread 1

Order ARIMA-Model: 212

| Forecast errors for |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | RMSE | in-sample | period |  |  |  |
| MAE | MPE | MAPE | MASE |  |  |  |
| Model 1 | 0.03213247 | 0.02222351 | 2.840216 | 39.55883 | 0.9242948 |  |
| Model 2 | 0.03211425 | 0.02221234 | 3.243932 | 39.66175 | 0.9238304 |  |
| Model 3 | 0.03210756 | 0.02219977 | 3.315172 | 39.48296 | 0.9233074 |  |
| Model 4 | 0.03213210 | 0.02222458 | 2.850158 | 39.62802 | 0.9243394 |  |
| Model 5 | 0.03211138 | 0.02220733 | 3.143650 | 39.57101 | 0.9236217 |  |
| Model 6 | 0.03212388 | 0.02222132 | 2.828735 | 39.65265 | 0.9242036 |  |
| Model 7 | 0.03213232 | 0.02222269 | 2.852097 | 39.57826 | 0.9242608 |  |
| Model 8 | 0.03213228 | 0.02222495 | 2.836466 | 39.59568 | 0.9243548 |  |

In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 3 & 6 & 3 & 3\end{array}$

Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model 10.019805180 .01347483345 .8247505 .82710 .9880384
Model 20.019844690 .01351324341 .3912496 .94880 .9908544
Model 30.019950100 .01365937370 .0649529 .61971 .0015697
Model 40.019812390 .01348789347 .7079507 .09850 .9889957
Model 50.019856380 .01356497352 .9571514 .16690 .9946473
Model 60.019773060 .01344331337 .4393493 .92190 .9857266
Model 70.019804490 .01347321347 .2408506 .32920 .9879192
Model $8 \quad 0.019810640 .01348587346 .2196506 .50390 .9888479$

Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}6 & 6 & 6 & 6 & 6\end{array}$

Figure 27: In-sample 1-step ahead forecast for Bund Spread 2


Figure 28: ARIMAX models: Forecast errors for Spread 2

Order ARIMA-Model: 212

|  | RMSE | MAE | MPE | MAPE | MASE |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Model 1 | 0.04264081 | 0.02981789 | 24.51972 | 64.93835 | 0.8906101 |
| Model 2 | 0.04256071 | 0.02974920 | 24.79284 | 64.98835 | 0.8885585 |
| Model 3 | 0.04250348 | 0.02978933 | 30.40421 | 69.85325 | 0.8897573 |
| Model 4 | 0.04262312 | 0.02979358 | 25.50014 | 64.54829 | 0.8898842 |
| Model 5 | 0.04258927 | 0.02980309 | 27.05524 | 66.49195 | 0.8901680 |
| Model 6 | 0.04260874 | 0.02978793 | 25.29414 | 65.03840 | 0.8897154 |
| Model 7 | 0.04262509 | 0.02979279 | 25.11718 | 64.85747 | 0.8898605 |
| Model 8 | 0.0426399 | 298195 |  |  |  |

In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 2 & 1 & 4 & 2\end{array}$

Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $1 \quad 0.018677650 .01366076292 .2786397 .74990 .8170953$
Model 20.018575930 .01362219309 .9956405 .73770 .8147882
Model 30.018700160 .01371103318 .2917417 .29100 .8201018
Model 40.018554660 .01346121303 .3802400 .37460 .8051595
Model 50.018824770 .01373133302 .5221406 .73130 .8213159
Model 60.018626060 .01351864299 .5526399 .70370 .8085942
Model 70.018580500 .01346084303 .6030402 .33740 .8051373
Model 80.018670220 .01366839291 .9199396 .75180 .8175514

Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}4 & 7 & 8 & 8 & 7\end{array}$

Figure 29: In-sample 1-step ahead forecast for Bund Spread 3


Figure 30: ARIMAX models: Forecast errors for Spread 3

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $1 \quad 0.028131360 .019144545 .24175523 .955010 .9343494$
Model 20.028106720 .019153365 .49447424 .329990 .9347798
Model 30.028091930 .019113475 .38982124 .333340 .9328333
Model 40.028119310 .019157485 .15578023 .949030 .9349812
Model 50.028105540 .019126665 .53649924 .493370 .9334771
Model 60.028122010 .019153825 .44417124 .234150 .9348023
Model $700.028122800 .019152275 .179378 \quad 23.90610 \quad 0.9347266$
Model 80.028128150 .019147855 .21671424 .004730 .9345111
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 3 & 4 & 7 & 3\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model 10.014761470 .009923028 -15.448704 359.97340 .8845386
Model $20.014996600 .010083953-13.724165383 .94250 .8988835$
Model $30.014985320 .010188350 \quad 3.515632396 .05160 .9081894$
Model $40.014861670 .009994870-18.934800369 .39730 .8909426$
Model $50.014987960 .010156042 \quad 5.913461393 .77330 .9053095$
Model $60.014914210 .010056793-12.036378384 .43040 .8964625$
Model $7 \quad 0.014807230 .009963518-20.068364367 .49620 .8881479$
Model $80.014808960 .009940353-14.514772360 .50340 .8860830$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}1 & 1 & 3 & 1 & 1\end{array}$

Figure 31: In-sample 1-step ahead forecast for Bund Spread 4


Figure 32: ARIMAX models: Forecast errors for Spread 4

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $10.028721130 .01941466-1.81247810 .575630 .9300209$
Model $20.028686270 .01941906-1.82422910 .632020 .9302316$
Model 30.028664800 .01938077 -1.823905 10.586480 .9283972
Model $40.028694090 .01942227-1.82401110 .630000 .9303852$
Model $50.028706040 .01939291-1.80303610 .564430 .9289787$
Model $60.028714310 .01941222-1.81111410 .584480 .9299040$
Model $7 \quad 0.028694250 .01942483-1.82368910 .629480 .9305079$
Model 80.028721520 .01941371 -1.808846 10.573750 .9299754
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 3 & 5 & 5 & 3\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model 10.014885950 .011084515 .349864171 .53570 .8870765
Model 20.015108300 .011177796 .409129170 .67460 .8945419
Model 30.015167800 .011307526 .291465170 .63540 .9049237
Model 40.014990110 .011118665 .639947171 .51020 .8898097
Model 50.015082850 .011270113 .822869172 .19030 .9019302
Model 60.015004930 .011201294 .882325172 .76960 .8964228
Model 70.014977610 .011106395 .752612171 .33060 .8888276
Model 80.014880060 .011079655 .430413171 .50910 .8866879
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}8 & 8 & 5 & 3 & 8\end{array}$

Figure 33: In-sample 1-step ahead forecast for Bund Spread 5



Figure 34: ARIMAX models: Forecast errors for Spread 5

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $10.045517120 .03251135-19.0393931 .554860 .8640701$
Model $20.045298560 .03214814-17.8132030 .158820 .8544169$
Model $30.045217590 .03211923-17.4709829 .855240 .8536484$
Model $40.045397630 .03228208-18.3179730 .700520 .8579765$
Model $50.045471970 .03245610-18.4826130 .962730 .8626016$
Model $60.045478090 .03244220-18.5103230 .979460 .8622322$
Model $7 \quad 0.045410980 .03230322-18.3301930 .738130 .8585386$
Model 80.045507740 .03249451 -19.02922 31.523570 .8636225
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 3 & 3 & 3 & 3\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $1 \quad 0.026412240 .02042963-73.67076213 .77560 .7824412$
Model 20.026309250 .02023777 -70.71468 213.43710 .7750927
Model $30.026401540 .02020716-71.70896213 .69370 .7739205$
Model 40.026042670 .02000847 -71.40281 215.43970 .7663109
Model $50.026410730 .02059951-64.33285207 .64770 .7889473$
Model $60.026434830 .02061408-64.12746208 .08750 .7895053$
Model $7 \quad 0.025989770 .01996735-72.51309214 .54010 .7647362$
Model $80.026454900 .02046102-72.82301214 .61890 .7836432$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}7 & 7 & 6 & 5 & 7\end{array}$

Figure 35: In-sample 1-step ahead forecast for Bund Spread 6



Figure 36: ARIMAX models: Forecast errors for Spread 6

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $1 \quad 0.023161010 .01592237$-2.649828 9.9562630 .8865528
Model 20.023139250 .01591927 -2.645949 9.933144 0.8863801
Model 30.023143370 .01592416 -2.629698 9.9319100 .8866522
Model $40.023146440 .01592203-2.6445389 .9332280 .8865335$
Model $50.023157390 .01592526-2.6479729 .9514640 .8867135$
Model $60.023168210 .01590048-2.57863719 .9434710 .8853335$
Model $7 \quad 0.023150670 .01592176-2.65762419 .9507660 .8865186$
Model 80.023156190 .01592284 -2.642216 9.9365640 .8865787
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}2 & 6 & 6 & 3 & 6\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model 10.018519200 .01381328 -14.19691 53.859500 .8389190
Model $20.018503550 .01382844-13.9978854 .117530 .8398393$
Model $30.018544160 .01387216-14.0249654 .269870 .8424947$
Model $40.018519050 .01383761-14.1398254 .133010 .8403966$
Model $50.018534110 .01383506-14.0873654 .080140 .8402418$
Model $60.018535270 .01384673-14.2193253 .860900 .8409502$
Model $70.018547690 .01386346-14.2743754 .060320 .8419666$
Model $80.018483200 .01378223-14.0674953 .933540 .8370328$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}8 & 8 & 2 & 1 & 8\end{array}$

Figure 37: In-sample 1-step ahead forecast for Bund Spread 7


Figure 38: ARIMAX models: Forecast errors for Spread 7

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $10.026859100 .01867746-4.61748320 .054600 .8378633$
Model $20.026518210 .01863829-4.22577019 .582180 .8361063$
Model $30.026474870 .01866500-4.25076419 .619520 .8373046$
Model $40.026523150 .01863054-4.19986019 .566430 .8357586$
Model $50.026755850 .01868481-4.61605819 .965830 .8381931$
Model $60.026840130 .01865309-4.59575420 .034620 .8367701$
Model $7 \quad 0.026850070 .01866583-4.64768320 .024280 .8373419$
Model $80.026529980 .01863080-4.16952019 .586410 .8357703$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 4 & 8 & 4 & 4\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $10.019304890 .01482760-3.88336318 .212120 .8297289$
Model 20.019256000 .01502088 -3.744408 18.439220 .8405449
Model $30.019370480 .01501833-3.74156118 .443040 .8404020$
Model $40.019270520 .01502890-3.74197618 .428450 .8409935$
Model $50.019497090 .01492010-3.93209818 .353870 .8349055$
Model $60.019107970 .01466240-3.80846617 .994590 .8204850$
Model $7 \quad 0.019335990 .01481477$-3.944982 18.200220 .8290114
Model $80.019251630 .01504934-3.68984418 .450380 .8421374$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}6 & 6 & 8 & 6 & 6\end{array}$

Figure 39: In-sample 1-step ahead forecast for Bund Spread 8


Figure 40: ARIMAX models: Forecast errors for Spread 8

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model 10.023261890 .0165010085 .63757130 .38770 .8217934
Model 20.022941100 .0164910074 .81666119 .57100 .8212954
Model 30.022961460 .0164625572 .61697117 .34720 .8198786
Model 40.022965940 .0164595171 .65120116 .33460 .8197273
Model 50.023245870 .0164932487 .00405131 .65220 .8214073
Model 60.023257820 .0164882485 .30132130 .02670 .8211579
Model $7 \quad 0.023257860 .01648575 \quad 86.32872131 .0196 \quad 0.8210342$
Model 80.022978250 .0164611874 .12761118 .82400 .8198104
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}2 & 4 & 4 & 4 & 4\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $10.020170490 .01623334-4.24733917 .792140 .8122666$
Model $20.020072690 .01643304-4.16245118 .091980 .8222589$
Model $30.019932950 .01637552-4.10015317 .997830 .8193806$
Model $40.019957110 .01639607-4.10664718 .022030 .8204093$
Model $50.020177750 .01630004-4.28331617 .840910 .8156040$
Model $60.020066650 .01617921-4.17783917 .714550 .8095579$
Model $7 \quad 0.020206830 .01622563-4.29438717 .796390 .8118809$
Model $80.019856000 .01641822-4.07580818 .002180 .8215176$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}8 & 6 & 8 & 6 & 6\end{array}$

Figure 41: In-sample 1-step ahead forecast for Bund Spread 9



Figure 42: ARIMAX models: Forecast errors for Spread 9
Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE
Model $1 \quad 0.020048990 .0147740210 .18904964 .571000 .8116739$
$\begin{array}{lllllll}\text { Model } 2 & 0.01994812 & 0.01478464 & 7.142526 & 61.21201 & 0.8122574\end{array}$
Model $3 \quad 0.019967830 .01476147 \quad 5.21403561 .072960 .8109844$
$\begin{array}{lllllll}\text { Model } 4 & 0.02002026 & 0.01480431 & 5.538086 & 60.33921 & 0.8133383\end{array}$
Model $50.019975700 .01481612 \quad 7.84578762 .638410 .8139872$
Model $600.019975270 .01477422 \quad 6.51867062 .171210 .8116851$
Model $7 \quad 0.020048960 .0147722710 .11543164 .546970 .8115778$
Model $8 \quad 0.020020470 .01480649 \quad 5.691398 \quad 60.399300 .8134582$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}2 & 3 & 3 & 4 & 3\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $1 \quad 0.019669670 .01617032-2.26688116 .547230 .8390916$
Model $20.019669260 .01617263-2.17818716 .513850 .8392117$
Model $30.019865430 .01625674-2.25195916 .641290 .8435763$
Model $40.019633290 .01614817-2.23155416 .523760 .8379425$
Model $50.019683720 .01616485-2.23658216 .509800 .8388078$
Model $60.019506170 .01608269-2.19638616 .426630 .8345446$
Model $7 \quad 0.019678040 .01618122-2.26607316 .557870 .8396573$
Model $80.019621490 .01613401-2.23267316 .508520 .8372076$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}6 & 6 & 2 & 6 & 6\end{array}$

Figure 43: In-sample 1-step ahead forecast for Bund Spread 10



Figure 44: ARIMAX models: Forecast errors for Spread 10

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE
Model $10.021123420 .01621805-39.68612119 .50920 .7981631$
Model 20.020663360 .01603717 -39.04804 122.72730 .7892613
Model $30.020919280 .01613474-41.21531124 .71120 .7940632$
Model $40.020929040 .01616055-41.74694125 .27830 .7953331$
Model $50.020834980 .01618949-38.64048118 .88370 .7967572$
Model $60.020711500 .01613586-38.69777118 .10400 .7941180$
Model $7 \quad 0.021120220 .01620052-39.34063119 .54900 .7973005$
Model $80.020931370 .01618113-42.04010125 .29940 .7963459$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}2 & 2 & 5 & 6 & 2\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $10.017326410 .01352390-11.3837531 .525250 .7464720$
Model $20.017719490 .01425832-10.9068032 .494260 .7870096$
Model $30.017626890 .01375413-11.1397731 .848810 .7591799$
Model $40.017469880 .01362539-11.1723731 .598430 .7520743$
Model $50.017427430 .01418379-11.2424332 .608370 .7828956$
Model $60.017350430 .01385035-11.2232732 .037560 .7644909$
Model $70.017353730 .01345079-11.3459731 .324750 .7424365$
Model $80.017445030 .01368609-11.2037631 .766450 .7554245$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}1 & 7 & 2 & 7 & 7\end{array}$

Figure 45: In-sample 1-step ahead forecast for Bund Spread 11


Figure 46: ARIMAX models: Forecast errors for Spread 11

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE
Model 10.037474660 .02448748 -1.480277 22.122260 .9243553
Model $20.037418050 .02447120-2.24275422 .558360 .9237408$
Model $30.037411070 .02448796-2.57157322 .919280 .9243734$
Model 40.037445790 .02448891 -1.581116 22.046220 .9244091
Model $50.037427570 .02445623-3.22245123 .811390 .9231756$
Model $60.037459020 .02447105-2.18087022 .747620 .9237352$
Model $7 \quad 0.03744661 \quad 0.02448614-1.522318 \quad 22.004760 .9243048$
Model $80.037474100 .02448958-1.53122322 .158020 .9244346$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}3 & 5 & 1 & 7 & 5\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model 10.027145110 .0203404310 .09445125 .630060 .8476207
Model 20.027314050 .0206086910 .27607225 .986300 .8587993
Model 30.027510940 .0207975110 .12109725 .974270 .8666679
Model $4 \quad 0.027244190 .02052319 \quad 9.843970 \quad 25.524400 .8552367$
Model $50.027224650 .0204797910 .232602 \quad 25.832260 .8534282$
Model 60.027003240 .0202802010 .06199025 .534100 .8451107
Model $7 \quad 0.027274980 .02053080 \quad 9.887745 \quad 25.571960 .8555539$
Model 80.027121700 .0203390010 .05503325 .590500 .8475610
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}6 & 6 & 4 & 4 & 6\end{array}$

Figure 47: In-sample 1-step ahead forecast for Bund Spread 12



Figure 48: ARIMAX models: Forecast errors for Spread 12

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $10.053741890 .02698553-2.40787713 .910840 .9498730$
Model $20.053711230 .02705320-2.44808013 .934620 .9522548$
Model $30.053720390 .02699165-2.43810813 .912050 .9500884$
Model $40.053740240 .02699927-2.41085313 .925900 .9503564$
Model $50.053705960 .02704891-2.41876513 .965220 .9521036$
Model $60.053706890 .02707558-2.42473113 .968530 .9530425$
Model $7 \quad 0.053740250 .02699991-2.41355413 .924430 .9503791$
Model 80.053741880 .02698388 -2.406356 13.91002 0.9498149
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}5 & 8 & 8 & 8 & 8\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model 10.027062630 .020403116 .64151923 .265840 .8116201
Model 20.027387950 .020507556 .77423823 .505350 .8157746
Model 30.027279940 .020528256 .88026023 .595570 .8165981
Model 40.027052360 .020368546 .69986823 .289790 .8102448
Model 50.027491290 .020572566 .77587023 .573330 .8183603
Model $6 \quad 0.027477180 .02057527 \quad 6.701891 \quad 23.494430 .8184684$
Model 70.027043300 .020364466 .69004823 .276090 .8100825
Model 80.027065160 .020404596 .64308423 .268150 .8116789
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}7 & 7 & 1 & 1 & 7\end{array}$

Figure 49: In-sample 1-step ahead forecast for Bund Spread 13



Figure 50: ARIMAX models: Forecast errors for Spread 13

```
Order ARIMA-Model: 2 1 2
Forecast errors for in-sample period
    RMSE MAE MPE MAPE MASE
Model 1 0.03320634 0.02301497 -267.1275 276.1111 0.9229970
Model 2 0.03310889 0.02310848 -219.6502 228.6838 0.9267473
Model 3 0.03313208 0.02310811 -223.6803 232.7105 0.9267323
Model 4 0.03317094 0.02305017 -246.7065 255.7055 0.9244089
Model 5 0.03318772 0.02303533 -266.1085 275.1066 0.9238138
Model 6 0.03318526 0.02304322 -265.7505 274.7545 0.9241299
Model 7 0.03320284 0.02301311 -267.2059 276.1870 0.9229225
Model 8 0.03317353 0.02304976 -246.6596 255.6582 0.9243923
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
    2 
Forecast errors for out-of-sample period
                            RMSE MAE MPE MAPE MASE
Model 1 0.02873870 0.02149838 -1.564736 21.05080 0.8571706
Model 2 0.02910143 0.02185049 -1.558878 21.41467 0.8712095
Model 3 0.02886032 0.02166511 -1.563774 21.23376 0.8638183
Model 4 0.02864411 0.02153057 -1.581835 21.05510 0.8584537
Model 5 0.02895465 0.02160563 -1.584694 21.17131 0.8614465
Model 6 0.02903370 0.02166041 -1.597657 21.20894 0.8636308
Model 7 0.02872098 0.02145591 -1.545458 21.02228 0.8554772
Model 8 0.02864837 0.02156104 -1.598899 21.07375 0.8596685
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
    4 4
```

Figure 51: In-sample 1-step ahead forecast for Bund Spread 14



Figure 52: ARIMAX models: Forecast errors for Spread 14

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $10.029642320 .02055978-4.23309021 .440710 .9041822$
Model $20.029585200 .02058376-3.85349221 .050850 .9052372$
Model 30.029611220 .02057088 -3.854051 21.093450 .9046707
Model 40.029614780 .02057869 -3.959893 21.213680 .9050141
Model $50.029615700 .02055517-4.24340321 .371740 .9039795$
Model 60.029611580 .02054991 -4.232424 21.386610 .9037482
Model $7 \quad 0.029639260 .02055249-4.21229421 .451850 .9038619$
Model $80.029617140 .02058863-3.97442521 .203670 .9054512$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}2 & 6 & 2 & 2 & 6\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $10.026698990 .02106374-4.64022119 .554540 .8099840$
Model $20.026904440 .02104527-4.68139519 .592410 .8092738$
Model $30.026650950 .02100066-4.62641919 .520930 .8075583$
Model $40.026598650 .02092262-4.61703519 .447970 .8045575$
Model $50.026967390 .02115561-4.70109719 .647050 .8135167$
Model $60.027073710 .02126188-4.72272419 .751590 .8176031$
Model $7 \quad 0.026693230 .02103895-4.64585119 .528780 .8090306$
Model 80.026603290 .02093859 -4.611764 19.465930 .8051714
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}4 & 4 & 8 & 4 & 4\end{array}$

Figure 53: In-sample 1-step ahead forecast for Bund Spread 15



Figure 54: ARIMAX models: Forecast errors for Spread 15

Order ARIMA-Model: 212
Forecast errors for in-sample period
RMSE MAE MPE MAPE MASE

Model $10.031540180 .02274253-25.34777104 .15560 .8562207$
Model $20.031194280 .02275400-19.33379105 .24060 .8566526$
Model 30.031243650 .02277991 -20.63031 104.77320 .8576280
Model $40.031243720 .02277955-20.60280104 .77930 .8576145$
Model $50.031526820 .02274257-25.30791104 .41920 .8562222$
Model $60.031539690 .02274136-25.31865104 .19950 .8561765$
Model $7 \quad 0.031527220 .02272087-25.34321104 .16000 .8554054$
Model $80.031256950 .02280143-20.62199104 .79240 .8584380$
In-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}2 & 7 & 2 & 1 & 7\end{array}$
Forecast errors for out-of-sample period
RMSE MAE MPE MAPE MASE
Model $10.027016310 .02179020-3.10080917 .136270 .7707261$
Model $20.027206710 .02198940-3.08967917 .423270 .7777719$
Model $30.026976010 .02192729-3.02429817 .356720 .7755753$
Model $40.026968940 .02191935-3.02388817 .350540 .7752942$
Model $50.027152170 .02186740-3.11059017 .202610 .7734567$
Model $60.027043890 .02180405-3.10726517 .147630 .7712162$
Model $7 \quad 0.027018050 .02185624-3.10656517 .188650 .7730620$
Model $80.026963130 .02183063-3.01806517 .275980 .7721563$
Out-of-sample: Model with lowest forecast error
RMSE MAE MPE MAPE MASE
$\begin{array}{lllll}8 & 1 & 8 & 1 & 1\end{array}$

